

Electromagnetic form factors of the $\Delta(1232)$ in dual-large N_c QCD*

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ABSTRACT: The three electromagnetic form factors of the $\Delta(1232)$ resonance, $G_M^*(q^2)$, $G_E^*(q^2)$, and $G_C^*(q^2)$ are obtained in the space-like region using a Dual Resonance Model realization of QCD in the large N_c limit (Dual-QCD $_\infty$). Each form factor involves a single free parameter which is fixed by fitting data on $G_M^*(q^2)$, and on the ratios $R_{EM}(q^2)$ and $R_{SM}(q^2)$. Good agreement with experiment is obtained for all three quantities. Results are then used to predict the q^2 -dependence of the chiral effective-field theory form factors $g_M(q^2)$, $g_E(q^2)$, and $g_C(q^2)$.

KEYWORDS: Phenomenological Models, QCD.

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1. Introduction

Quantum Chromodynamics (QCD) in the limit of infinite number of colours (QCD_∞) [1] is known to be solvable, predicting a hadronic spectrum made of an infinite number of zero-width resonances [2]. However, the hadronic parameters in this spectrum (masses, couplings, etc.) remain unspecified, so that a model is required to fix them. Some models of this spectrum have been proposed for heavy quark Green's functions [3]–[4], as well as for light quark systems [5]. The infinite number of zero-width resonances of QCD_∞ evokes Veneziano's Dual-Resonance model [6], the precursor of string theory. In fact, drawing from this model, a concrete realization of QCD_∞ has been proposed, namely Dual-QCD_∞ [7]–[8]. In this framework the masses and couplings in three-point Green's functions are chosen so that form factors are expressed as an Euler Beta function of the Veneziano type. The asymptotic Regge behaviour of these form factors in the space-like region is power-like, and controlled by a single free parameter of the model. In the time-like region Dual-QCD_∞ is not afflicted by unitarity violations known to affect n-point functions ($n \geq 4$). In fact, three-point functions can be safely unitarized by simply shifting the poles from the real-axis into the second Riemann sheet in the complex energy (squared) plane [7, 9]–[10]. This procedure also provides an estimate of the corrections to QCD_∞ expected from the fact that $N_c = 3$. These corrections are small, and of order $\Gamma/M \lesssim 10\%$, where Γ and M are a typical width and hadronic resonance mass, respectively.

It is perhaps important to emphasize that Dual-QCD_∞ is simply a model realization of QCD_∞ which fixes the otherwise unspecified masses and couplings in this theory. Hence, it is not a model based on the $1/N_c$ expansion. In fact, N_c is assumed infinite from the start, and no attempt is made to obtain corrections from a power series expansion in $1/N_c$. For other model realizations of QCD_∞, but based on the same philosophy, see e.g. [11].

Dual-QCD_∞ has been applied with great success to the pion form factor $F_\pi(q^2)$ in both the time-like and the space-like regions [7, 12], as well as to the proton form factors in the latter region [8]. For the pion one finds excellent agreement with data in the wide range $-q^2 = 0 - 10 \text{ GeV}^2$, resulting in a chi-squared per degree of freedom $\chi_F^2 \simeq 1.1$. A

similar high quality fit is also achieved in the time-like region. In the case of the proton, the results for the form factors $F_1(q^2)$ and $F_2(q^2)$, or $G_M(q^2)$ and $G_E(q^2)$, are in very good agreement with data in the available range $-q^2 = 0 - 30 \text{ GeV}^2$. Furthermore, once the free parameter in each form factor is fixed by the fit, there follows a prediction for the ratio $G_E(q^2)/G_M(q^2)$. This ratio is in reasonable agreement with polarization transfer data indicating a strong deviation from the so-called scaling law ($G_E(q^2)/G_M(q^2) \simeq \text{const.}$). It is important to mention that if the free parameter of the Dual-QCD $_\infty$ form factor is determined from a fit to the high $-q^2$ data, one subsequently achieves agreement all the way down to $q^2 = 0$. The mean-squared radius then becomes a prediction, which e.g. in the case of the pion agrees with data to within one-third of a standard deviation. Clearly, the converse procedure can be applied with equivalent results, i.e. fixing the free parameter from the radius leads to agreement with space-like data everywhere.

While Dual-QCD $_\infty$ is a particular realization of QCD $_\infty$, it can also be viewed as an Extended Vector Meson Dominance model providing corrections to single rho-meson dominance. These corrections arise from the contributions of the radial excitations of the rho-meson. In fact, as is well known, experimentally $F_\pi(q^2)$ falls off with $-q^2$ faster than a monopole, and $G_M(q^2)$ faster than a dipole. In Dual-QCD $_\infty$ this is precisely the case, the source of the correction being the radial excitations of the rho-meson. At this point it should be mentioned that Perturbative QCD (PQCD) together with some empirical counting rules [13] lead to monopole and dipole type of asymptotic behaviour for $F_\pi(q^2)$ and $G_M(q^2)$, respectively. However, these results are believed to hold only at extreme asymptotic momenta. In fact, while deep-inelastic scattering indicates precocious scaling, this does not seem to be the case for exclusive processes, such as elastic scattering, or semi-inclusive reactions such as tau-lepton decay into hadrons.

In the past few years there has been a renewed interest in understanding the electromagnetic structure of the $\Delta(1232)$, largely motivated by high precision measurements of the photon induced $N \rightarrow \Delta(1232)$ transition at electron beam laboratories (LEGS, BATES, ELSA, MAMI, and J-LAB) [14]. Concurrently, on the theory sector, lattice QCD and several dynamical models, as well as chiral effective-field theories, have been used to confront the data [14]. Motivated by the success of Dual-QCD $_\infty$ in accounting for the data on the pion and proton form factors, we perform here a determination in this framework, and in the space-like region, of $G_M^*(q^2)$, $G_E^*(q^2)$, and $G_C^*(q^2)$, the so-called Jones-Scadron [15] electromagnetic form factors of the $\Delta(1232)$. After fixing each one of the three free parameters from fits to data, a prediction is obtained for the chiral effective-field theory form factors $g_M(q^2)$, $g_E(q^2)$, and $g_C(q^2)$.

2. Form factors in dual-QCD $_\infty$

In the literature there are quite a few conventions for the three electromagnetic form factors of $\Delta(1232)$. Some of these are affected by kinematical singularities, and not all are dimensionless. We choose here the Jones-Scadron definition [15] in which all three form factors are free of kinematical singularities, and are dimensionless. Generally, in the

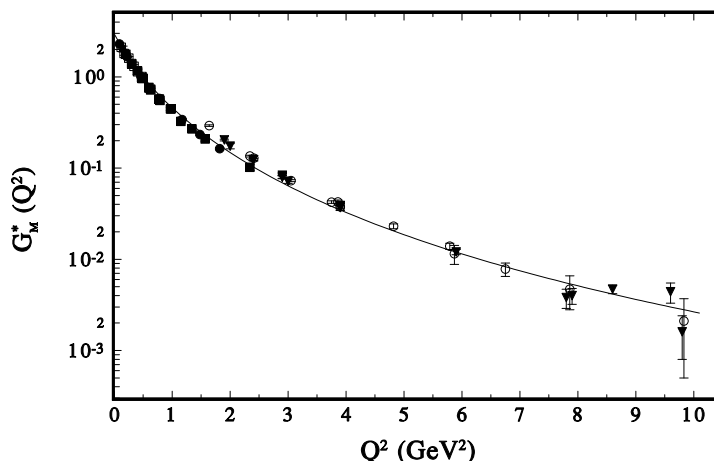


Figure 1: The magnetic form factor $G_M^*(Q^2)$, eq. (2.5), with $\beta_M = 4.6$, together with the data [19].

framework of QCD_∞ one expects the form factors to be given by

$$G_i^*(s) = \sum_{n=0}^{\infty} \frac{C_{in}}{(M_n^2 - s)}, \quad (2.1)$$

where $i = M, E, C$ correspond to M (magnetic dipole), E (electric quadrupole), and C (Coulomb quadrupole), respectively, $s \equiv q^2$, and the masses of the vector-meson zero-width resonances, M_n , as well as their couplings C_{in} , are not predicted in this framework. In Dual- QCD_∞ these are chosen so that the form factors are given by Euler Beta functions, i.e.

$$C_{in} = G_i^*(0) \frac{\Gamma(\beta_i - 1/2)}{\alpha' \sqrt{\pi}} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\beta_i - 1 - n)}, \quad (2.2)$$

where $\beta_i (i = M, E, C)$ are free parameters controlling, respectively, the asymptotic behaviour of the form factors $G_M^*(Q^2)$, $G_E^*(Q^2)$, $G_C^*(Q^2)$ in the space-like region ($s < 0$), and $\alpha' = 1/2M_\rho^2$ is the universal string tension entering the rho-meson Regge trajectory

$$\alpha_\rho(s) = 1 + \alpha'(s - M_\rho^2). \quad (2.3)$$

The mass spectrum is chosen as [16]

$$M_n^2 = M_\rho^2(1 + 2n). \quad (2.4)$$

Using eqs. (2.2) and (2.4) in eq. (2.1) one obtains

$$\begin{aligned} G_i^*(s) &= G_i^*(0) \frac{\Gamma(\beta_i - 1/2)}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\beta_i - 1 - n)} \frac{1}{[n+1 - \alpha_\rho(s)]} \\ &= G_i^*(0) \frac{1}{\sqrt{\pi}} \frac{\Gamma(\beta_i - 1/2)}{\Gamma(\beta_i - 1)} B(\beta_i - 1, 1/2 - \alpha' s), \end{aligned} \quad (2.5)$$

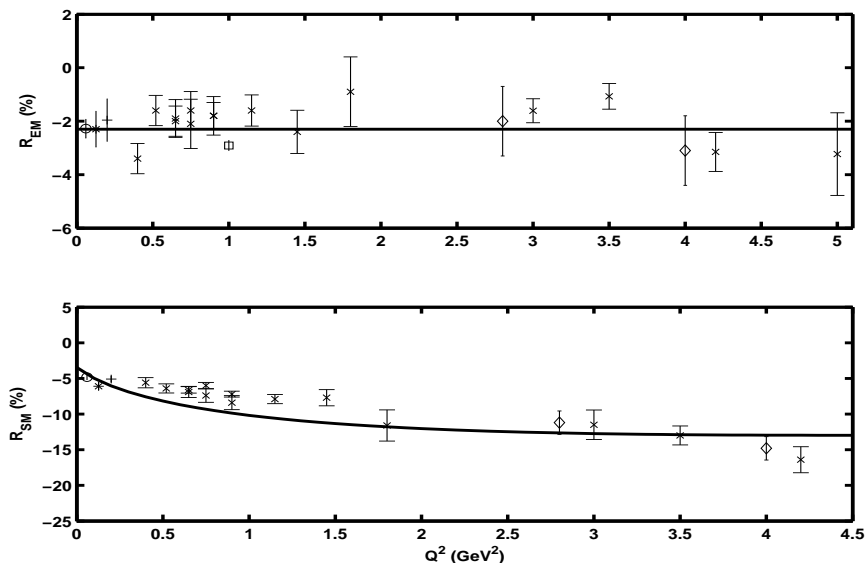


Figure 2: The ratios $R_{EM}(Q^2)$ and $R_{SM}(Q^2)$ defined in eq. (3.1) with $\beta_M = \beta_E = 4.6$, and $\beta_C = 6.2$, together with the data [20].

where $B(x, y) \equiv \Gamma(x) \Gamma(y)/\Gamma(x + y)$ is the Euler Beta function. In the time-like region ($s > 0$) the poles of the Beta function correspond to an infinite set of zero-width resonances with equally spaced squared masses given by eq. (2.4). In fact, from eq. (2.5) it follows

$$Im G_i(s) = G_i^*(0) \frac{\Gamma(\beta_i - 1/2)}{\alpha' \sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n + 1)} \frac{1}{\Gamma(\beta_i - 1 - n)} \pi \delta(M_n^2 - s). \quad (2.6)$$

Asymptotically, the Regge behaviour of the form factors in the space-like region is power-like, i.e.

$$\lim_{s \rightarrow -\infty} G_i(s) = (-\alpha' s)^{(1-\beta_i)}, \quad (2.7)$$

The free parameters β_i can be fixed from fits to the data in this region. Notice that the values $\beta_i = 2$ reduce the form factors to single rho-meson dominance (naive Vector Meson Dominance).

The mass formula eq. (2.4) predicts e.g. for the first three radial excitations: $M_{\rho'} \simeq 1340$ MeV, $M_{\rho''} \simeq 1720$ MeV, and $M_{\rho'''} \simeq 2034$ MeV in reasonable agreement with experiment [17]: $M_{\rho'} = 1465 \pm 25$ MeV, $M_{\rho''} = 1700 \pm 20$ MeV, and $M_{\rho'''} = 2149 \pm 17$ MeV. Alternative, e.g. non-linear, mass formulas have been proposed in an attempt to match the asymptotic Regge behaviour to the Operator Product Expansion of current correlators at short distances [18]. However, the differences with linear formulas in the values of the first few masses are at the level of a few percent. Hence, the form factors are hardly modified, since the contribution from higher resonances is factorially suppressed, as seen from eq. (2.5).

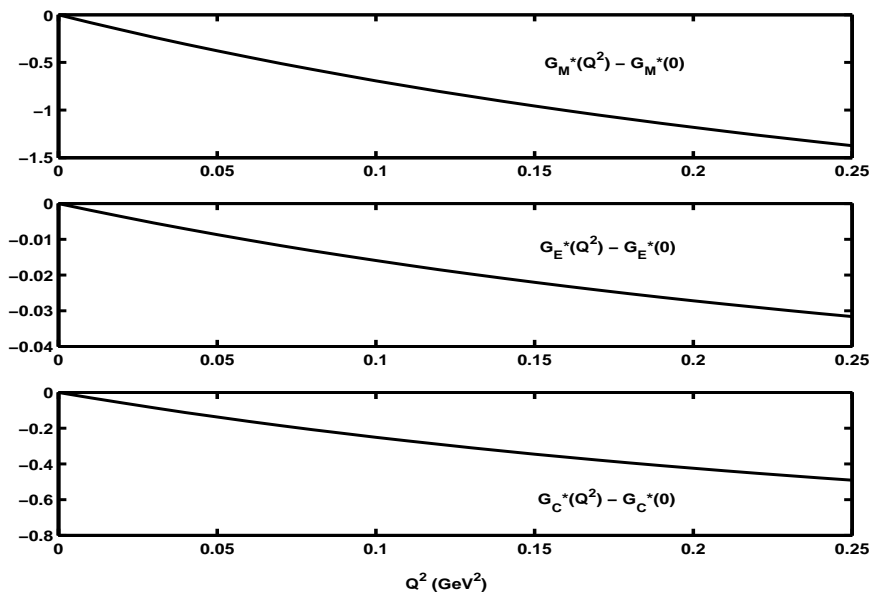


Figure 3: The differences $G_i^*(Q^2) - G_i^*(0)$, ($i = M, E, C$), for $\beta_M = \beta_E = 4.6$ and $\beta_C = 6.2$.

3. Results

In order to fix the free parameter in each of the three form factors we use data on $G_M^*(q^2)$, and on the ratios

$$R_{EM} = -\frac{G_E^*(Q^2)}{G_M^*(Q^2)}, \quad R_{SM} = -\frac{Q_+ Q_-}{4 M_\Delta^2} \frac{G_C^*(Q^2)}{G_M^*(Q^2)}, \quad (3.1)$$

where

$$Q_\pm^2 = (M_\Delta \pm M_N)^2 + Q^2, \quad (3.2)$$

and in the sequel we use $Q^2 = -q^2 = -s \geq 0$. The normalization of the form factors $G_i^*(Q^2)$ is [14]: $G_M^*(0) = 3.04$, $G_E^*(0) = 0.07$, and $G_C^*(0) = 1.00$. The ratios above are then normalized as: $R_{EM}(0) = -2.30\%$, and $R_{SM}(0) = -3.46\%$. The best fit to data on $G_M^*(Q^2)$ is achieved for $\beta_M = 4.6 - 4.8$. Figure 1 shows $G_M^*(Q^2)$ for $\beta_M = 4.6$ together with the experimental data [19]. The value $\beta = 4.8$ changes only slightly the high Q^2 tail of this curve. Next, we fit the ratio $R_{EM}(Q^2)$, eq. (3.1). Given the large experimental errors in this quantity it is not possible to go beyond the obvious fit $R_{EM}(Q^2) \simeq constant$, or $\beta_M \simeq \beta_E$. This result is shown in figure 2 for $\beta_M = \beta_E = 4.6$, together with the data [20]. Future improvement in the accuracy of this data might reveal some Q^2 -dependence in this ratio, which would require slightly different values of β_E .

Finally, the ratio $R_{SM}(Q^2)$, eq. (3.1), can be fitted with $\beta_M = 4.6$ and $\beta_C = 6.0 - 6.2$. This is shown in figure 2 together with the data [20]. In figure 3 we show the differences $G_i^*(Q^2) - G_i^*(0)$, ($i = M, E, C$), which can be compared with a similar figure given in [14] showing predictions from several dynamical models [21]–[22]. Our results appear to agree qualitatively and quantitatively with most of these models.

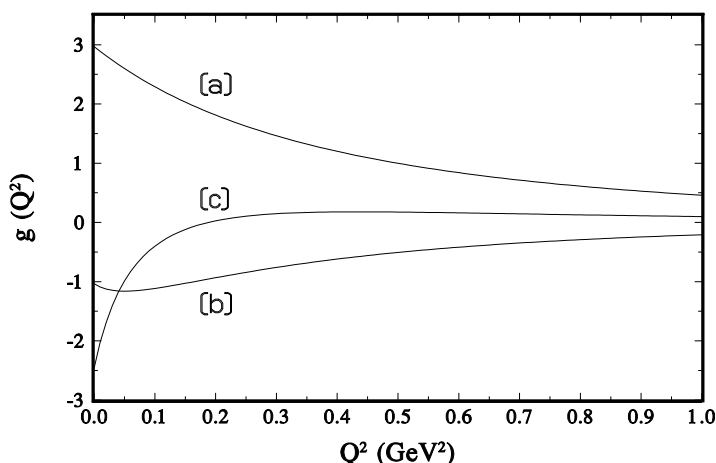


Figure 4: The chiral effective-field theory form factors $g_M(Q^2)$ (curve [a]), $g_E(Q^2)$ (curve [b]), and $g_C(Q^2)$ (curve [c]), at low Q^2 .

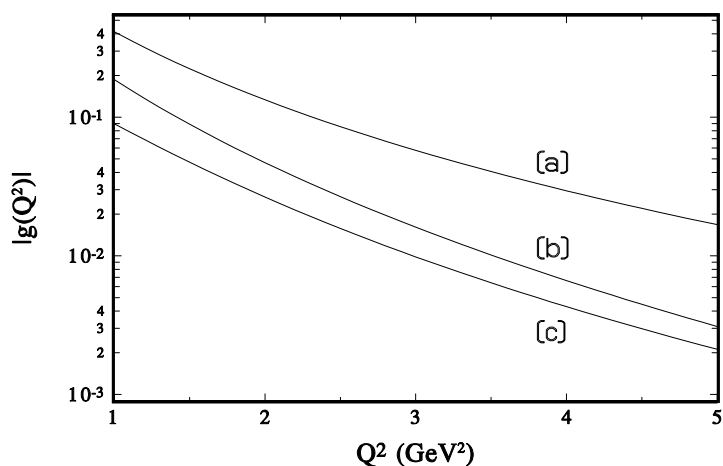


Figure 5: The absolute values of the chiral effective-field theory form factors $|g_M(Q^2)|$ (curve [a]), $|g_E(Q^2)|$ (curve [b]), and $|g_C(Q^2)|$ (curve [c]), at intermediate Q^2 .

There is an alternative set of electromagnetic form factors of the $\Delta(1232)$, which are being used in the framework of a chiral effective-field theory [22], namely $g_M(Q^2)$, $g_E(Q^2)$, and $g_C(Q^2)$. These are related to the Jones-Scadron form factors as

$$g_M(Q^2) = G_M^*(Q^2) - G_E^*(Q^2), \quad (3.3)$$

$$g_C(Q^2) = Q_+^2 \frac{[\mu(Q^2) G_C^*(Q^2) + 4 M_\Delta^2 G_E^*(Q^2)]}{\mu^2(Q^2) + 4 M_\Delta^2 Q^2}, \quad (3.4)$$

$$g_E(Q^2) = 2 \frac{[Q_+^2 G_E^*(Q^2) - Q^2 g_C(Q^2)]}{\mu(Q^2)}, \quad (3.5)$$

where $\mu(Q^2) \equiv M_N^2 - M_\Delta^2 + Q^2$. These form factors are normalized as [14]: $g_M(0) = 2.97$, $g_E(0) = -1.00$, and $g_C(0) = -2.60$.

Predictions for these form factors are shown in figure 4 at low Q^2 , and as absolute values in figure 5 at intermediate Q^2 , both for $\beta_M = \beta_E = 4.6$ and $\beta_C = 6.2$ in the Jones-Scadron form factors.

4. Conclusions

Motivated by the good results obtained for the pion and proton electromagnetic form factors in Dual-QCD $_\infty$ [7]–[8], this framework has been used here to parametrize the three Jones-Scadron electromagnetic form factors of the $\Delta(1232)$ in the space-like region. These form factors have the advantage of being dimensionless and free of kinematical singularities [15]. The single free parameter in each form factor has been fixed by fitting experimental data on $G_M^*(Q^2)$ and on the ratios $R_{EM}(Q^2)$, and $R_{SM}(Q^2)$. Very good agreement with the data is achieved for $G_M^*(Q^2)$ in the wide range of momentum transfers $Q^2 = 0 - 10 \text{ GeV}^2$. For $\beta_M \simeq 4.6$ in eq. (2.5), $G_M^*(Q^2)$ falls off with Q^2 faster than a dipole. In fact, from eq. (2.7) one finds $G_M^*(Q^2) \sim (\alpha' Q^2)^{-3.6}$. This follows the trend that starts with the pion form factor, which falls off faster than a monopole, $F_\pi(Q^2) \sim (\alpha' Q^2)^{-1.3}$, and the proton form factors, which fall off faster than a dipole, $F_1(Q^2) \sim (\alpha' Q^2)^{-2.03}$, $F_2(Q^2) \sim (\alpha' Q^2)^{-3.2}$. Future data on $G_M^*(Q^2)$ at higher Q^2 might require some fine tuning on the value of β_M . Current data on the ratio $R_{EM}(Q^2)$, needed to fix β_E , extends only up to $Q^2 \simeq 5 \text{ GeV}^2$. Within the rather large errors, this ratio is consistent with a constant, or $\beta_E \simeq \beta_M$. The ratio $R_{SM}(Q^2)$, though, does show an appreciable Q^2 -dependence leading to $\beta_C = 6.2$. The values found for all the three parameters β_i carry an uncertainty at the level of a couple of a percent.

Having fixed the Jones- Scadron form factors, $G_i^*(Q^2)$, predictions then follow for an alternative set of form factors, $g_i(Q^2)$, used in a chiral effective-field theory [22]. These predictions should be of interest in this framework as they provide simple analytical expressions for the form factors in a wide range of momentum transfers.

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